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Statistical Hair on Black Holes

Andrew Strominger

Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08855
and
Department of Physics, University of California, Santa Barbara, CA 93106

Abstract

The Bekenstein-Hawking entropy for certain BPS-saturated black holes in string theory has recently been derived by counting internal black hole microstates at weak coupling. We argue that the black hole microstate can be measured by interference experiments even in the strong coupling region where there is clearly an event horizon. Extracting information which is naively behind the event horizon is possible due to the existence of statistical quantum hair carried by the black hole. This quantum hair arises from the arbitrarily large number of discrete gauge symmetries present in string theory.

More than twenty years ago Bekenstein [1] and Hawking [2] showed that black holes possess a macroscopic thermodynamic entropy equal to one quarter the area of the event horizon in Planck units: $S_{BH} = A/4$. This beautiful result cried out for a microscopic statistical derivation. However, because the Planck length appears in the relation it is likely that a quantum theory of gravity is required for such a derivation. In the last several months it has been found that string theory - a candidate for a quantum theory of gravity - can provide a precise derivation of this entropy in a variety of circumstances [3-18].

The derivation exploited the existence of two distinct descriptions of extremal BPS-saturated states in $N = 4$ or $N = 8$ supersymmetric string theories. The first description is as a semiclassical quantum state corresponding to the usual extremal Reissner-Nordström solutions[1]. The second description is as a quantum bound state of elementary D-brane solitons [19-21] and strings. The logarithm of the bound state degeneracy (as a function of the charges) as computed in the second picture agrees (for large charges) with S_{BH} as computed in the first.

These two pictures are physically relevant at different values of the string coupling g_s [3]. At strong coupling[2], string perturbation theory is divergent, fluctuations in the D-brane bound state configuration are large, and the D-brane description is inaccurate. Supersymmetric nonrenormalization theorems nevertheless protect the low energy effective action. Hence the black hole solution, which is independent of g_s, should still provide an accurate description of the state. In a slight abuse of terminology we refer to this region of couplings as the 'black hole phase' (we do not mean to suggest there is a sharp phase transition at some value of g_s). For very weak string coupling, string perturbation theory is good, and the D-brane picture is valid. However in this region the string length, which grows (in Planck units) as the coupling decreases, becomes larger than the Schwarzchild radius. The black hole is smaller than a typical string. At scales shorter than the string length string theory gives drastic modifications of our notion of Riemmanian geometry, even at the classical level. In particular there is no obvious notion of causality or an event horizon at such short scales. Hence the black hole picture is inaccurate in this 'D-brane phase'.

[1] Or generalizations thereof involving extra scalar fields or more dimensions, depending on the context.

[2] By which we mean $g_s \sim 1$. For $g_s >> 1$ the theory may have a weakly coupled dual description.

There is no overlapping range of validity of the two pictures (at least in examples considered so far). The agreement between the entropies as computed in the two pictures was nevertheless expected because, in an $N = 4$ or $N = 8$ supersymmetric theory the number of BPS states typically does not change as a function of the coupling [22]. This fact enables one to extrapolate the D-brane phase calculation into the black hole phase. Such extrapolations will certainly not be possible for general S-matrix elements in the theory, which will depend strongly on the coupling.

One would like to apply the new insight from string theory into black hole entropy to the black hole information puzzle. One form of the argument that information is lost is briefly as follows. Throw a neutral string in a definite quantum state into a large extremal black hole. The black hole then becomes nonextremal and acquires a nonzero Hawking temperature. It reradiates back to extremality via Hawking emission. In the semiclassical regime the outgoing quanta arises from a pair production process outside the horizon, and hence is blind to the internal quantum state of the black hole. Hence the final state of extremal black hole + outgoing quanta is insensitive to some aspects of the initial state. Unitarity is violated and information is lost. Further discussions of this example, including subtleties which could invalidate the conclusion that information is lost, can be found in [23], [24].

This puzzle exists only in the black hole phase. In the D-brane phase there is no event horizon and hence no reason to expect that the outgoing quanta cannot access information about the incident quanta. Indeed the scattering is explicitly calculable and perturbatively unitary. One point of view [25], is that the system is like a neutron star. Everyone would agree that the quantum state of a neutron star can be measured in principle. However if the gravitational coupling is turned up, the neutron star will collapse into a black hole at a critical value of the coupling. Then by the no-hair theorem it would appear that no information about the quantum state is available. Hence the fact that there is perturbatively unitary scattering in the D-brane phase does not immediately imply that there is no unitarity violation in the black hole phase: there could be some kind of collapse at a critical value of g_s.

Clearly in order to address the information puzzle we must analyze the black hole phase. Herein we address the simplest question: Can the quantum state of an extremal black hole be measured[3] in the black hole phase? In the D-brane phase the state can

[3] As with any single quantum object, the state may change during the measurement process and we can at most hope to determine the quantum state after the measurement is completed.

clearly be measured by scattering strings off of the bound state. However we do not know how to compute string scattering in the black hole phase, and the semiclassical no-hair theorem suggests that the scattering depends only on a few charges and is insensitive to the specific black hole microstate.

A simple argument shows that the answer to this question is nevertheless yes, assuming the existence of a low-energy effective field theory on scales large compared to the size of the black hole[4]. In such a theory black holes are effectively nonrelativistic, pointlike quantum particles. In the zero-charge sector of the Hilbert space, black holes are absent: The low-energy effective field theory involves only the massless fields. However we may also consider the charged sector of the Hilbert space, for which the groundstate is in general degenerate and corresponds to a static collection of BPS-saturated black holes. Low energy excitations above such charged groundstates include non-relativistic motion of the black holes, and are also describable by effective field theory. In the D-brane phase this effective field theory contains one (super)field for every quantum state of the black hole:

$$\mathcal{S}_{eff} \sim -\sum_{i=1}^{e^{A/4}} \int d^4x((\nabla\Phi_i)^2 + m^2(\Phi_i)^2 + ...) \tag{1}$$

Each field Φ_i creates a BPS-saturated supermultiplet. At low energies above the groundstate, only non-relativistic modes of these fields contribute, and (1) reduces to the quantum mechanics of slowly-moving black holes, as discussed below. The mass m may depend on g_s and other moduli but is highly constrained by supersymmetry. As mentioned above, in an $N = 4$ or $N = 8$ theory the number of such states typically does not change as a function of g_s. Hence (1) should be valid in the black hole phase as far as the number of fields. Determining all of the interactions however could involve solving an intractable strong-coupling problem.

It follows immediately from these assumptions, without any knowledge of the interactions, that the quantum state can be measured. Given that the states are described by an effective action of the form (1), their identity can be determined from interference experiments. For example in the scattering of two objects there is interference between Feynman

[4] The reader may object that this amounts to assuming the answer. If this assumption truly led to a contradiction with semiclassical reasoning we might be forced to question it. However our point, made below, is that it *is* compatible with semiclassical reasoning, and hence there is no reason to suspect its validity.

diagrams which differ by the exchange of the final state objects if and only if they are identical. These experiments can be performed at low energies and large volumes, where effective field theory is valid. It is also possible, by appropriate choice of the initial state, to avoid bringing the objects near to each other and the complicating effects of bound states at threshold. Repitition of such experiments will enable one to ascertain which of a large collection of like-charged black holes are identical and which are not.

This conclusion might at first seem to be in conflict with semiclassical reasoning. Semiclassical low-energy scattering of two BPS black holes with identical charges reduces to quantum mechanics on the two-black-hole moduli space (after any high-energy modes of the Φ_i fields are integrated out). This moduli space can apparently be reliably computed from the known classical two-black-hole solutions. It depends only on the charges and not on the quantum state of the black hole, which does not enter into the classical solutions. Hence the scattering would appear to be independent of the quantum state.

In fact there is an error in this reasoning. The classical moduli space is ambiguous due to a singularity on the subspace where the positions of the two black holes coincide. There is a Z_2 symmetry which exchanges the two black holes and acts freely on the moduli space everywhere except at the coincident points. Because of this singularity one cannot determine from the classical solutions whether the moduli space is topologically $\mathcal{M}_2 \sim R^3 \times R^3$ or $\mathcal{M}_2 \sim (R^3 \times R^3)/Z_2$. If we do divide by the Z_2, the quantum mechanics will give interference, and otherwise not. So the the semiclassical scattering in the black hole phase cannot be be unambiguously determined form low-energy field theory, and the possibility of interference is *not* in conflict with low-energy reasoning.

In string theory the moduli space can be unambiguously determined. At weak coupling the the D-brane picture tells us to divide by Z_2 if the black holes are in the same state, and otherwise not. However the topology of the moduli space cannot change as g_s is varied, so this tells us that the moduli space should also be a Z_2 quotient in the black hole phase. This is somewhat reminiscent of Callan-Rubakov electron-monopole scattering: The low-energy effective theory has an ambiguity concerning the boundary condition at the origin, which can only be resolved by consideration of the high-energy unified gauge theory. More generally for a large collection of N like-charged BPS black holes there is a discrete permutation symmetry S_N which acts freely everywhere except at points corresponding to one or more coincident black holes. Hence classically the moduli space is determined only up to division by an arbitrary discrete subgroup of S_N,*i.e.* there is an ambiguity concerning whether or not the exchange of two identically charged black holes

returns one to the same point in the multi-black hole moduli space. All of these ambiguities are resolved by going to weak coupling and using the D-brane picture, from which we are instructed to take the quotient by any element of S_N which exchanges black holes in the same quantum state. Once the identifications are determined, the moduli space is known and it is possible to determine directly from the moduli space which black holes are in the same quantum state and which are not[5].

In conclusion, the quantum state of the black hole can be measured in the black hole phase. This does *not* contradict semiclassical reasoning because of an ambiguity in the semiclassical moduli space. Resolution of this ambiguity requires the data which determines the quantum state.

Some time ago it was observed that black holes can carry quantum hair which is measurable by interference experiments in which strings lasso black holes [26]. This indicated that classical solutions do not contain all the information about the low-energy quantum behavior of black holes. Similarly here we see another example in which classical solutions do not determine low-energy quantum behavior of black holes, leading to statistical quantum hair. Statistical hair was first considered in [27].

In the present context, statistical hair arises from discrete gauge symmetries. When two identical D-branes coincide, there is an enhanced $SU(2)$ gauge symmetry. The separation between the two D-branes corresponds to a vev for a scalar field in the adjoint of $SU(2)$ living on the D-branes. Exchanging the two D-branes corresponds to reversing the sign of the vev, which is a discrete Z_2 gauge transformation. More generally S_N arises as the Weyl group of an enhanced $SU(N)$ gauge symmetry. Hence the moduli space identifications arise from unbroken discrete gauge symmetries. Although the context is different, this suggests a connection with the discrete gauge hair discussed in [28]. In [29] it was found that discrete gauge hair has exponentially small but measurable effects on the state of the quantum fields outside of the black hole. (Similar effects were found for axion hair

[5] Further dependence of the moduli space on the quantum state arises when one considers the possible formation of bound states at threshold, which is proportional to wave function overlaps and hence suppressed at large volume. These bound states correspond to lower-dimensional branches of the moduli space which emanate from regions corresponding to one or more coincident black holes. There are many such branches labeled by the quantum microstate of the bound states. The connectivity of these various branches again depends on the microstate and not just the charges.

in string theory in [30].) It would be interesting to see if statistical hair also leads to such effects.

Some time ago it was speculated that black holes in string theory carry so much quantum hair that it is possible to completely reconstruct the initial state which formed the black hole, and that this might somehow lead to a resolution of the information puzzle [31] [29]. One problem with this idea was that, according to our understanding of string theory and quantum hair at the time, there seemed to be only a finite number of types of quantum hair which could encode only a finite amount of information. However we now know that string theory has an arbitrarily large number of such unbroken discrete gauge symmetries, since there is one for every pair of D-branes and there can be arbitrarily large numbers of D-branes. Hence statistical hair can provide an arbitrarily large amount of information about a black hole, unlike previously discussed types of quantum hair. For this reason it is tempting to speculate that statistical hair may play a key role in the string theoretic resolution of the black hole information puzzle.

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